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13. ABSTRACT *(Maximum 200 words)*

We conducted a fundamental study to investigate the dynamic behavior of laminated structures. Both finite element and analytical solutions were developed. Using a 48 degree-of-freedom shell finite element (FE) developed by Kapania and coworkers, the effect of arbitrary inplane and out-of-plane loads on the transverse vibrations of thin arbitrarily laminated panels with or without geometric imperfections was analyzed. The element was used to investigate the effect of geometric imperfections on geometrically nonlinear impact response of thin laminated plates and cylindrical panels using reduced basis methods. A FE model was also developed to study the linear and geometrically nonlinear transient behavior of laminated beams. A post-processor was developed that can use the transverse displacements from a classical laminated plate theory (CLPT) or a first order shear deformation theory (FSDT) to determine the transverse normal and shear stresses. The nonlinear dynamic equations of the first-order shear deformation theory and the third-order shear deformation plate theory of Reddy was reformulated to describe the interior and edge zone problems of rectangular plates. A three-dimensional elasticity solution for the free vibration analysis of general cross-ply laminated plates has been carried out by combining Fourier series solution with a state space representation and the transfer matrix approach. A close theoretical examination have clarified the accuracy of the various single layer theories and a layerwise theory with the exact elasticity solution. Analytical results have been obtained for a series of laminated plates subjected to low-velocity impact using various theories. A key feature of the approach developed here is that both the impact force and the resulting responses are obtained simultaneously.

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FINAL REPORT
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STUDY OF WAVE PROPAGATION AND DYNAMIC RESPONSE OF LAMINATED COMPOSITE STRUCTURES

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1. SUMMARY

Comosites are often used in situations involving the sudden application of loads. In this work, we conducted a fundamental study to investigate the dynamic behavior of laminated structures. Both finite element and analytical solutions were developed. Using a 48 degree-of-freedom shell finite element developed previously by Kapania and coworkers, the effect of arbitrary inplane and out-of-plane loads on the transverse vibrations of thin arbitrarily laminated panels with or without geometric imperfections was analyzed. The same element was used to subsequently investigate the effect of geometric imperfections on geometrically nonlinear impact response of thin laminated plates and cylindrical panels using reduced basis methods. A finite element model was also developed to study the linear and geometrically nonlinear transient behavior of laminated beams. A post-processor was developed that can use the transverse displacements from a classical laminated plate theory (CLPT) or a first order shear deformation theory (FSDT) to determine the transverse normal and shear stresses. The nonlinear dynamic equations of the first-order shear deformation theory and the third-order shear deformation plate theory of Reddy was reformulated to describe the interior and edge zone problems of rectangular plates. Viscous damping terms were also included. Efforts were also made to analytically study the impact problem of laminated composite plates. A three-dimensional elasticity solution for the free vibration analysis of general cross-ply laminated plates has been carried out by combining Fourier series solution with a state space representation and the transfer matrix approach. A close theoretical examination have clarified the accuracy of the various single layer theories and a layerwise theory with the exact elasticity solution. Numerical results (for various displacements and stress components) have been obtained for a series of laminated plates subjected to low-velocity impact using various theories. A key feature of the approach developed here is that both the impact force and the resulting responses are obtained simultaneously.

This study resulted in a large number of publications in archival journals, conference presentations, research reports, and doctoral dissertations.

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2. INTRODUCTION

Composite materials are the materials of the future. The wide usage of these materials for space, military and commercial applications at present reflect the increased interest in composite materials. The main advantages of using composites lie in their high strength and stiffness to weight ratios. In addition to these advantages, laminated composite materials can be tailored to design requirements by varying the laminate scheme.

Composites are used in situations involving the sudden application of loads. The dynamic response of the structure ensues after load application and a state of stress may be generated that leads to failure. It is necessary to understand the response characteristics of the material body for all important effects, including geometry, boundary conditions, and loading.

Investigations in the area of wave propagation in laminated medium had been conducted by geologists and physicists interested in the study of the wave propagation of seismic waves. The increasing use of laminated composites in the aerospace, automotive and naval structures has led to a more elaborate area of research. These structures are subject to high velocity impact by birds, meteoroids, and undersea animals. New theories, experiments and numerical techniques must be developed for an extensive study of nonlinear impact response and wave propagation in laminates.

During the construction of composite structures or the fabrication of composite structural components, substantial deviations between the actual and intended shapes may occur. These geometric imperfections can significantly alter the structural behavior, namely, static response, free and forced vibration response, and buckling and postbuckling behavior. In addition, the response of composite structures to impact loads has been a problem of considerable interest and concern in recent years since laminated structures are vulnerable to a foreign object impact even in low velocity. The impact response of laminated composite plates subjected to a foreign object in low velocity has been investigated extensively by experimental and numerical methods. Most of the numerical studies are based on analytical investigations and have been performed without considering the nonlinear effects and geometric imperfections of the plates. The transient elastic response of plates subjected to arbitrary transient loads is an important design consideration for laminated structures that are of interest to the U. S. Army. These plates may undergo moderately large deflections of the order of the plate thickness under extreme dynamic loadings. Thus it is very important to include nonlinear effects in the analysis of composite structures subjected to impact loads.

The transient response of laminated plates subjected to impact loads was investigated by analytical and numerical methods. Goldsmith [1] used the normal modes method to

determine the dynamic response of an isotropic plate or beam to a rigid impactor. Timoshenko [2] used normal modes and a Hertzian contact law to analyze the deflections of a beam impacted by an impactor. The resulting nonlinear integral equations were solved by numerical integration. Sun and Chattopadhyay [3] extended Timoshenko's method to a laminated simply supported composite plate. Ramkumar and Chen [4] used Fourier integral transforms to find the response of an infinite anisotropic laminated plate to an experimentally determined impact force. Petersen [5] used the finite element method based on a shear deformable plate theory with rotary inertia to analyze laminated plates subjected to impact loads. Thangjitham et al. [6] obtained low-velocity impact response of orthotropic plates using a higher-order theory which incorporates the transverse normal stress and rotary inertia effects and fulfills the shear stress conditions on the bounding surfaces. Sun and Liou [7] used a three-dimensional hybrid stress finite element method to investigate laminated plates under impact loads. Cairns and Lagace [8] obtained transient response of graphite/epoxy and Kevlar/epoxy laminates subjected to impact using a Rayleigh-Ritz method. Chao, Tung, Sheu, and Chern [9] employed a 3-D laminated theory in conjunction with a modal analysis to study the impact response analysis of thick comoposites. Mal and Lih [10,11] performed a 3-D elastodynamic reponse anlysis of thick composites under point and line loads.

Poe [12] performed a combined experimental-theoretical study to simulate the behavior of laminates under low-velocity impact. The impact was simulated quasi-statically by pressing spherical indentators against the laminates. The fibers directly under the impact site were found to be broken. The theretical study employed theory of elasticity (a semi-infinite isotropic body under pressure on part of the boundary) and maximum stress criterion to obtain the length and depths of the cracks under quasi-static loads. Experimental studies have been also performed by Wu and Springer [13], Joshi [14], Cordell and Sjöblom [15] and Gong and Sankar [16]. Detailed reviews of various investigations into impact response and wave propagation studies have been given by Kapania and Raciti [17] and Abrate [18].

For the study of impact response of metals and composites, many researchers [1-8] used the Hertzian contact law which relates impactor and plate motion with contact force. However, Yang and Sun [19] showed that the Hertzian contact law was not adequate by performing statical indentation tests on graphite/epoxy composite laminates using spherical steel indenters of different sizes. They found that significant permanent indentations existed. In order to account for the permanent indentation, Tan and Sun [20] proposed a modified Hertzian contact law following Yang and Sun [19]. They compared experimental results with the predictions of finite element analysis using the statically determined contact law. Sun and Chen [21] analyzed initially stressed composite plates under impact loads using

this modified Hertzian contact law. It is noted that Bogdanovich and Yarve [22] proposed a method which combined calculation of stress-strain state in a laminated plate on the basis of spline-approximation of displacements. A variational approach was used for studying the process of impact contact interaction between the indentor and the plate. This method [22] was subsequently extended to the calculation of damage zones in laminated composite plates subjected to low velocity impact [23]. Sankar and Sun [24] used plane stress finite elements to study the low-velocity impact of laminated beams subjected to initial stresses.

Most of the impact problems have been formulated using the small deflection theory which is adequate if the impact load is small. However, if a plate undergoes large deflections of the order of the thickness, it is necessary to include the geometric nonlinearity. Chen and Sun [25] investigated nonlinear transient response of composite laminates subjected to impact loads with initial stresses. They used the finite element method based on the Mindlin plate theory in conjunction with an experimentally established contact law [20]. Kant and Mallikarjuna [26] used a higher-order theory and C^0 finite elements to analyze a laminated plate under impact loads using the modified Hertzian contact law by Tan and Sun [20].

In the various studies on the impact response of laminated plates, the effect of geometric imperfections has not been included (although some research on impact response of curved panels have been performed recently [27–31]). The geometric imperfections are known to significantly change the free vibration and dynamic response behavior of structures. Watawala and Nash [32] studied the free and forced motions (due to base excitation caused by seismic effects) of imperfect circular cylindrical shell. It was concluded by these authors that the geometric nonlinearity plays a significant role in the dynamic response of a cylindrical shell if the displacements are equal to or greater than one half of the shell thickness.

However, current nonlinear impact models based on the finite element method [25,26] may take several hours of CPU time on large computers to analyze one impact scenario. It is well known that a lot of computational effort is needed to solve the nonlinear transient problem. Considerable effort has been devoted to reducing the cost and/or time of solving nonlinear problems. One way to approach the problems is by developing computational algorithms which solve a reduced set of equations instead of solving the original equations. These methods are known as reduction methods since a reduced set of equations is solved to get the original system response. The reduction of the problem size is achieved by approximating the vector of discrete fundamental unknowns by a linear combination of a small number of global approximation functions called the basis vectors. Several basis vectors are used for the nonlinear transient analysis.

Damage in composite laminates can be classified into three major categories, namely,

(1) transverse cracking, (2) delamination, and (3) fiber breakage [13,14, 33–37]. All these damage modes are not separate, but inter-connected and act concurrently leading to the degradation of mechanical and other properties of composite laminate. In the last decade considerable progress has been made in understanding the onset and growth of various damage modes. But, this understanding is limited to the cases of simple geometry and loading conditions. Besides, much of the research is directed towards understanding the effect of a single damage mode such as matrix cracking [38–44] and delamination [45–49] on the stiffness/strength of the laminate. But, in reality, all the damage modes are present simultaneously and act concurrently in degrading the performance of the laminate.

A long term goal is to predict the combined effect of various damage modes and external environment on the degradation and failure of the laminate. However, the existing literature survey [50–54] shows that not much work has been done to understand and quantify the combined effect of these damage modes on the performance of the laminate, whereas, this understanding is essential for efficient design and production of damage resistant and durable composite laminates and structures. Hence, there is a practical need for developing a general tool to predict the combined effect of various damage modes on the performance of the composite structures with complex geometry and loading conditions.

For the solution of nonlinear dynamic problems, Horri and Kawahara [55] proposed the use of the mode superposition method. They solved nonlinear dynamical problems of simple structures subjected to harmonic excitation. Eigenvalues and eigenvectors used as basis vectors were updated at every time increment. The use of the mode superposition method was also done by Nickell [56], Morris [57] and Remseth [58]. The principle behind the use of mode superposition method for nonlinear dynamic problem is that small harmonic motion may be superimposed on large static motion, and that small forced motion may be represented in terms of the nonlinear (tangent stiffness) frequency spectrum. Similar approach was used by Shah, et al. [59] and Bathe and Gracewski [60]. But they used mode shapes and natural frequencies of a initial stiffness matrix and all nonlinearities are included in the evaluation of internal force vector corresponding to element stresses. Recently Mohraz, et al. [61] used the mode superposition method to compute the inelastic dynamic response of frame structures. Clough and Wilson [62] used Ritz vectors in conjunction with substructure analysis for dynamic analysis of structures with local nonlinearities. Noor [63] used initial and steady state eigenvectors for the case of step loading on arches and spherical shells. The steady state eigenvector is obtained by first finding the steady state (or static) nonlinear solution and using it to evaluate the stiffness matrix of the structure, then extracting the lowest vibration modes from the corresponding generalized eigenvalue problem. Efforts to

combine eigenvectors or Ritz vectors and their derivatives with respect to reduced generalized coordinates together as basis vectors were made by some researchers. Dominant eigenvectors of the tangent stiffness matrix and their derivatives were used by Idelsohn and Cardona [64]. Ritz vectors and their derivatives were used by Idelsohn and Cardona [65] and Chang and Engblom [66]. In both cases, the use of derivatives enhanced convergence rate and accuracy of solutions. Das, et al. [67] proposed a reduced basis method, derived from the path derivatives method of Noor [63,68], for studying the inverse dynamics of large space cranes. In their suggested method, the basis vectors are chosen from the deflections at various time steps during the initial stage of the simulation (i. e. the response vectors obtained at successive time instants, separated by a small time intervals, during the initial stage) are used as the basis vectors for the rest of the simulation.

3. DESCRIPTION OF THE CURRENT RESEARCH

Composites are often used in situations involving the sudden application of loads. A dynamic response of the structure ensues after the load application and a state of stress may be generated that leads to failure. It is necessary to understand the response charecteristics of the material body for all important effects, including geometry, boundary conditions and loading. With this objective in mind, a research program under a grant from the Army Research Office (ARO) was initiated by the two PI's in June 1990. A brief description of some of the research carried out to date is given here.

Kapania and Byun [69] analyzed the effect of complex, arbitrary inplane and out-of-plane loads on the transverse vibrations of thin arbitrarily laminated panels with or without geometric imperfections using the finite element method. A 48 degree of freedom thin shell element previously formulated by Kapania and coworkers [70–72] was employed. The formulation is general with respect to the boundary conditions, types of imperfections, and number, orientation, and stacking sequence of lamina. The results were presented for angle-ply rectangular plates and cross-ply cylindrical panels,with both of these having simply supported edge conditions along all the four edges, and isotropic square plates involving free edges under nonuniform combined loads. The effects of geometric parameters and material properties were examined.

In subsequent studies, Kapania and Byun [73,74] investigated the effect of geometric imperfections on geometrically nonlinear impact response of thin laminated plates using the finite element method. In these studies, we also investigated the effectiveness of some of the currently available reduction methods in reducing the computer time required to obtain the geometrically nonlinear transient response under impact loads. Reduction methods using

two different types of expansion modes as the basis vectors were employed : (i) those based on eigenvectors for the system under consideration, and (ii) those based on Ritz vectors [75,76]. These methods were used with and without basis updating and the resulting responses were compared with those obtained by solving the full set of equations, the so-called direct solution [77,78]. Approximate solutions were obtained by integrating both the original full equations and the reduced set of equations using the Newmark ¢6nstant-acceleration method. It was observed that geometric imperfections cause significant changes in the impact response of thin laminated plates. The basis updating was found to be essential to obtain accutate results. When applied to the impact response problem, it was observed that a large number of basis vectors are needed to account for the high frequency charecteristics of the impact response. The reduction method used in this study were found to be only marginally better than the method that integrates the complete set of nonlinear equations (about 7 percent). The lack of improvement in the reduction methods was traced to the fact that most of the CPU time is spent in calculating the internal nodal force vector.

Obst and Kapania [79] developed a finite element to study the static, dynamic, linear and nonlinear responses of general laminated beams using Reddy's third-order shear deformation theory . Note that in Reddy's third-order shear deformation theory there are stress discontinuities in normal and shear stresses at the interfaces of any two adjacent layers. Kapania and Obst [80] developed a linear beam finite element using a recently developed theory [Savithri and Varadan, 81] for cross-ply laminated plates in which normal and shear stresses are forced to be continuous at the interfaces of any two adjacent layers.

In studying the impact response of laminated structures, it is important to consider damage, such as delamination, matrix cracking, or fiber breaks. To consider such damage, it is important to get very accurate information for in-plane and transverse stresses. However, finite element models based on the classical lamination plate theory (CLPT) or the first order shear deformation theory (FSDPT) can not give accurate interlaminar stresses from constitutive relations. A postprocessor for transverse normal and shear stresses as well as in-plane stresses was developed by Byun and Kapania [82] using the first order shear deformation theory and the CLPT. The transverse stresses were obtained by integrating elasticity equilibrium equations which include the influence of the product of in-plane stresses and out-of-plane rotations on the z-directional equilibrium equations. The post-processor was based on using global approximations of the displacement data obtained from finite element analysis. Two different types of orthogonal polynomials, Chebychev polynomials and orthogonal polynomials given by Ralston [83] were used in approximating the displacement data. Although, the Ralston polynomials were little more complicated to generate than

the Chebychev polynomials, the former enabled us to use arbitrary mesh in finite element programs and yielded more accurate results than the Chebychev polynomials. Using finite elements program based on both CLPT [72] and FSDPT [84], the transverse stresses were obtained for both symmetrical and unsymmetrically laminated plates. A good agreement was obtained with the 3-D elasticity results given by Pagano [85] for symmetrically laminated plates and those given by Chaudhuri and Seide [86] for unsymmetrically laminated plates. The present formulation did not include the inertia effects.

Research was also performed using analytical methods. Nosier and Reddy [87] reformulated the the nonlinear dynamic equations of the first-order theory and the third-order shear deformation plate theory of Reddy into equations describing the interior and edge zone problems of rectangular plates [88]. Viscous damping terms were also included. It was shown that, for certain boundary conditions, the number of governing equations can be reduced to three, as in the classical plate theory. The problems of static large deflection and dynamic small deflection of rectangular plates were considered. Numerical results were presented to demonstarate the effects of nonlinearity, shear deformation, rotatory inertia, damping and sonic boom type loadings. Nosier and Reddy [89,90] also performed analytical studies on the free vibration and buckling of symmetrically laminated plates (including those made of transeversely isotropic materials) using the interior and edge-zone equations associated with first and third-order shear deformation theories of Reddy [91] and Levinson [92]. A conclusion of this study was that the contribution of the edge-zone equations on the natural frequency and the buckling load was negligible.

Efforts were also made to analytically study the impact problem of laminated composite plates. The contact force is determined using the piece-wise linear Lagrange and Hermite Cubic functions. A contact law which incorporates the permanent identation is also taken into account. Excellent agreement has been observed for the case of the impact of an isotropic beam . A three-dimensional elasticity solution for the free vibration analysis of general cross-ply laminated plates has been carried out by combining Fourier series solution with a state space representation and the transfer matrix approach. A close theoretical examination have clarified the accuracy of the various single layer theories and a layerwise theory with the exact elasticity solution (see, Nosier, Kapania and Reddy [93]). Numerical results (for various displacements and stress components) have been obtained for a series of laminated plates subjected to low-velocity impact using various theories. Six models of varying complexity were employed to represent the spatial distribution of the impact force. While spatial distribution does not affect the overall displacement of the plate, it significantly influences the stress distribution (hence the damage) in the contact zone. A key feature of

the approach developed here is that both the impact force and the resulting responses are obtained simultaneously (see, Nosier, Kapania and Reddy [94]).

Although a large number of studies dealing with impact response of flat laminated panels has been conducted in the past, relatively little work seems to be available on impact of laminated cylindrical panels. Kapania and Stoumbos [95] presented an analytical study to predict the nonlinear transient response of thin imperfect laminated cylindrical panels subjected to impact loads. A review was also given regarding the state of the art on the impact response of laminated plates and shells. The imperfect panels were modeled using a 48 degree-of-freedom (DOF) finite element based on the classical laminated plate theory. Linear and nonlinear transient responses were obtained using the modal superposition method (MSM) and a reduction method based on Ritz vectors (R-R), both in conjunction with direct integration schemes (Wilson-θ or Newmark methods). The effect of the number of modes used was also discussed. A modified contact law was incorporated to evaluate the impact loads due to a projectile. Different non-dimensional shell radii $[r/h]$ were used in order to study the effect of the non-dimensional shell radii $[r/h]$ on the impact response of laminated cylindrical panels. The effect of geometric imperfections on linear and nonlinear transient response under sudden impact loads was also analyzed. Both MSM and R-R methods used in the analysis were found to have acceptable accuracy and the accuracy increased with the number of modes used. It was observed that the reduction of the non-dimensional shell radius $[r/h]$ causes significant changes in the impact response of the cylindrical panel: a significant decrease in the central deflection and contact force histories. The introduction of geometric imperfections led to similar conclusions.

Kapania and Stoumbos are currently conducting research towards studying accurate and efficient techniques for performing nonlinear impact response of composite structures. A code is currently being created to study the transient response of structures using explicit time integration schemes for a set of second-order nonlinear governing equations. The scheme will allow us to use different time steps in different sub-domains. This will be especially usefull for impact response studies which require very small elements in the vicinity of the impact. We will be able to use very small time steps in the domain in the vicinity of the impact and larger time steps away from the impact zone. The major problem will be the reconciliation of the response where the various zones meet. Efforts are being made to overcome these problems. In addition to the impact response studies, the method will also be applicable to other engineering structures which need highly nonuniform meshes or are composed of different materials. The method is usefull because a small or stiff element does not impose a restrictive time step on the entire mesh. This research will be a major component of Mr.

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